New Boxes from Old

Synopsis

Students take a rectangular box (*e.g.*, a cereal box) and cut it up to make a new, cubical box with the same volume as the original. In so doing, they will discover that because the cubical box has less surface area than the original, a cube is a more efficient way to package things. To display their work, students design and construct a mobile, to be hung from a ceiling at school or at home.

Objectives

Students will recognize that a cube has less surface area than a rectangular prism of the same volume. They will also gain valuable experience in problem solving with spatial-visual relationships.

Materials

The most important items needed for this exercise are, of course, boxes. Each student will need ***two identical*** boxes, which can be brought from home. At least ten days before doing this exercise, send home the following note, with the appropriate date and phone number filled in:

Needed: BOXES!

For a math activity on ____________________________, each student will need two identical, empty boxes. These boxes should have all six faces attached and be made of medium-weight cardboard or stiff paper.
Examples of boxes that will work well include: small or medium sized boxes that hold cereal, crackers, or pasta; and boxes containing macaroni and cheese, pudding or cake mixes; or boxes containing hot chocolate, tea, or instant oatmeal packets. Computer diskette boxes and boxes holding contact lens solutions will also work well.
However, shoe boxes or other boxes with separate lids, juice boxes, and drink cartons will not work well. Very small boxes will be difficult to work with, as will very large ones. Cube-shaped boxes will not work.
Thank you for your help, and feel free to call me at ________________ if you have any questions.

It is a good idea to collect several sets of boxes yourself for those students who forget or are unable to obtain a pair of identical boxes. In general, the more rectangular and less cube-like a box is, the easier it will be to make a cube-shaped box from it.

In addition to the boxes, you will need some ordinary classroom supplies. For each of these, try to have available as many as you can get, up to one of each per student.

- Scissors
- Rolls of tape, both masking and transparent
- Rulers
- Calculators
- Legal size envelopes or zipper-style plastic bags (quart or gallon size)
- Large paper clips

In order for students to display their old and new boxes together in a mobile, you will need:

- about twenty 36-inch dowel rods that are three-sixteenths of an inch in diameter (available in hardware stores and hobby shops for about 25 - 50 cents each)
- two or three utility knives (or inexpensive paring knives)

- one or two single-hole punches
- two or three spools of regular sewing or heavy thread
- a few bottles of white glue and/or clear nail polish
- several sheets of poster paper, in a variety of colors

Procedure

Students should know how to determine both the surface area and volume of a rectangular prism before starting this exercise. They should also have a clear understanding of cubes: in a cube all the dimensions are equal, so the volume of a cube is the length of any side raised to the third power, or cubed.

In order to create a cube-shaped box from a rectangular box, however, students will also have to be able to work backwards. For a cube of known volume, they will need to be able to figure out how to find its dimensions: if the volume of a cube is equal to its length cubed, the length of any side of a cube is equal to the cube root of its volume. Some simple examples will help illustrate this. Ask the class, for example, what the dimensions would be for a box with a volume of 8 cubic centimeters, or for a box whose volume is 27 cubic inches.

Then you can move on to harder examples. What if the volume of the box were 21 cubic inches? If students have graphing calculators such as the TI-82, they can find cube roots easily using the MATH functions key. If not, they will get some good practice with estimation and trial-and-error as they determine that the cube root of 21 is about 2.76. (When they make their own cube-shaped boxes, they will work in centimeters and millimeters, so cube roots need not be taken out beyond the nearest hundredth.)

Once students are clear on how to work surface area and volume problems, allow them a class period (40-45 minutes) to begin measuring, calculating, and taking apart one of their boxes, working from the first of the **Instructions for Students, New Boxes from Old**. Some students may be able to complete the construction of a new, cube-shaped box, but probably most will not. Use the paper clips to keep the parts of a box-in-progress together overnight, and have students store their scraps in the plastic bags, which also can be clipped to the box parts.

During the next class period, students should be able to complete the construction of their cube-shaped boxes, and create mobiles from their original boxes, new boxes, and scraps, working from the second set of **Instructions for Students, The Boxes Go Mobile**.

Making a well-balanced mobile can be much harder than it looks! Some students may lack the dexterity needed to tie knots in sewing thread, and they should be encouraged to use their peers for help. Often it is just a matter of needing another set of hands to hold a dowel rod steady while a knot is tied. Other students may experience frustration when they try to balance the different components on the mobile. Try to avoid doing too much for these students, since a lot of learning can occur as they puzzle over the relationships between the weights of the different components and where on the dowel the threads holding them need to be. Without realizing it, they are gaining valuable experience with levers, fulcrums, and loads.

Discussion and Extensions

Ask your students to share with the class their answers to the last question (#13) on the *New Boxes from Old* student pages. While the total areas of their scraps should equal the differences in surface areas of their two boxes, it is unlikely that they will actually be very close. Measurement inaccuracies, rounding, and the difficulties of cutting straight lines and right angles will all combine to make their answers not as closely matched as they ought to be.

Since we are all concerned about preserving natural resources, ask the class which type of packaging would use the least paper: selling pasta, cereal, crackers, and cake mix, etc., in rectangular boxes, or in cube-shaped boxes. They should notice by now that rectangular boxes can be downright wasteful. Ask them to look around at all the mobiles and note which types of boxes generated the most scraps relative to the sizes of the boxes. They should be able to notice that long, thin boxes, such as spaghetti boxes, had more scraps left over than boxes that had some faces that were square or nearly square, such as a diskette box. See if they can summarize their observations in mathematical terms, *e.g.*, "When the length to width and length to height ratios are close to one, there are fewer scraps than when one or both of these ratios is much greater (or less) than one."

Since a cube-shaped box uses less material, why don't companies sell cereal and other foods this way? Ask students to share their thoughts about this question. If they need help, ask them to imagine how they would arrange many boxes of food in the same cabinet. Wouldn't lots of items have to be two or three rows back in the cabinet, and wouldn't items be stacked in at least two layers? What if they wanted the box of cereal that was all the way in the back and on the bottom layer?

Then ask them to think about picking up that cube-shaped box and pouring some cereal out of it. Would they have to hold the box with two hands because the box is so wide? Would this be awkward? And would the box now need a special pouring spout in order to get the cereal into a bowl instead of all over the counter? (One major cereal company is currently experimenting with a milk carton-style package.)

Students might also realize that when a consumer walks down the cereal aisle of a grocery store, each cereal company wants the consumer to buy its type(s) of cereal. The companies, therefore, want nice, big areas on their boxes so they can attract the consumer's attention and advertise what's inside. A cube-shaped box, with its smaller area facing the consumer, might not be as eye-catching as the usual rectangular box.

Some foods, because of their particular shapes, require rectangular packages. Spaghetti and lasagna noodles, for example, would have to be cut short to fit into a one-pound, cube-shaped box. Otherwise, a cube-shaped box containing standard-length noodles (about 26 cm) would be quite large. Just for fun, you can have your students determine the number of spaghetti noodles such a box would hold, if its dimensions were equal to the length of a typical noodle. (The answer depends on whether the box is filled with thick or thin spaghetti. We counted 812 noodles in a one-pound box of thin spaghetti, which means there would be about 23,500 noodles in the cube-shaped box. Of course, there would be fewer noodles in a box of thick spaghetti.) It is also interesting to note how heavy such a cubical box of noodles would be (29 pounds) and to speculate on whether or not the thin cardboard used in pasta boxes would be strong enough to support this weight (not likely).

In our area there is a factory where the boxes that hold many consumer products are made, and we have been able to arrange tours of the plant for our classes after they have completed this exercise. Students and teachers alike have been amazed to see all the steps involved in designing, printing, cutting out, and assembling the boxes. If you can find a packaging company in your area (look in the yellow pages under *Packaging*, or call your local Chamber of Commerce), a field trip would be very worthwhile.

Instructions for Students

In this exercise, you will take one of your two identical boxes and cut it up to make a new box that is cube-shaped. Both boxes will have the same volume--but will they have the same surface areas? By the time you have finished making your new box, you should be able to answer this question.

The directions below will help you construct your cube-shaped box, after which you will be able to draw some conclusions about the relationships between volumes and surface areas of boxes. *PLEASE read through ALL the directions BEFORE beginning!!!*

1. Measure each dimension (length (*L),* width *(W),* and height (*H*)) of your box to the nearest millimeter.

L = *W* = *H* =

2. Calculate the surface area (*SA)* of your box.

SA =

3. Calculate the volume *(V)* of your box.

V =

Did you remember to include units in your answers?

4. Carefully open the glued edges of your box; it was probably made from one flat piece of cardboard. Cut off any parts of flaps that were hidden from view when the box was still intact. The hidden parts are usually easy to spot because they generally don't have any color on them and/or they do have dried glue on them. You can also look at your other box to make sure. *Please check with your teacher if you aren't sure what is okay to cut off!* After you've cut off these hidden parts, throw them away in the trash.

5. Using the volume you determined in step 3, calculate what the length of any side of your new cube-shaped box should be.

Length of any side =

6. Find someone else in the class who has just finished step 5. Trade papers and boxes and check each other's measurements and calculations. If your answers don't agree, work together until you decide which of you is correct.

7. On the inside of your opened-out box, draw the six identical squares you will need to make your cube-shaped box. Remember that their sides must all equal the length you calculated in step 5, and the sides must meet at 90° angles. You may find that you can only fit four or five complete squares on your opened-out box. If that is the case, you will have to take some of the remaining scraps and tape them together, rather like a jigsaw puzzle, to make the last one or two sides of your cube.

8. After you have figured out how to obtain all six sides of your cube, cut them out. *Important: save any remaining scraps!* Put them in an envelope or zipper-type plastic bag. (It's okay to fold them if you need to.)

9. As neatly as you can, tape the six squares together to form your cube-shaped box. It will be sturdy and look good if you use masking tape on the *inside* of the cube to attach adjacent squares and then use clear tape only on the *outside* for additional strength.

10. Calculate the surface area of your cube-shaped box.

SA of cube =

11. Compare this number to the surface area of the original box, which you determined in step 2. Are they the same?

If not, by how much do they differ?

Difference in surface areas =

12. Find the area of each of the scraps. Since some of them may be oddly-shaped, you may want to divide them into squares and rectangles that will be easier to measure and calculate areas for. After you have determined all of their areas, add them up to get one total area of the scraps.

Total area of scraps =

13. Compare this number to the number you got in step 11.

Are the two areas the same?

Should they be the same?

Why or why not?

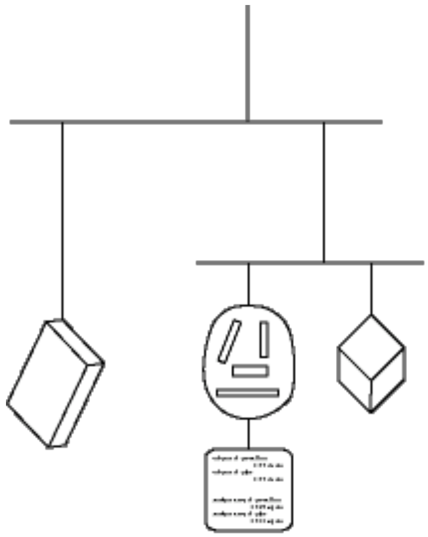
The Boxes Go Mobile

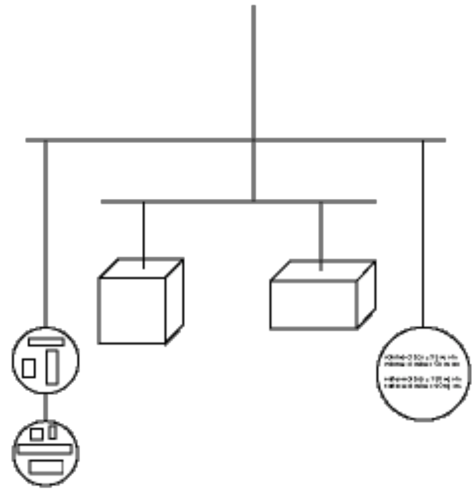
Now that you've made a cubical box from a rectangular one, you can create an artistic display of your work by making a mobile. A good-looking and interesting mobile requires some planning in advance, however, and patience is essential when it comes to actually hanging the individual components.

Your mobile should have four components. Components one and two are your boxes. Component three is your collection of scraps, artistically arranged and glued to poster paper. Be creative! You can cut the poster paper to any size and shape that will be visually appealing and appropriate to the size of the scraps you have.

The fourth component of your mobile is a mathematical comparison of the two boxes. This should be printed neatly or word-processed on white or light-colored paper, and then glued to another piece of poster paper. Again, you can cut the paper to an appropriate size and shape to make it visually appealing. Your comparison should list the pertinent information for each box, including the dimensions, volume, and surface area. It should also include the total area of the scraps.

Once you've got all four components together, you will need to decide how to arrange the components so they hang from sections of dowel rods *and* can be balanced so the dowels will hang horizontally. There are lots of ways to do this, so experiment and see what works and looks interesting. A few examples are shown below.





To cut a dowel rod to the length you need, make a pencil mark where you want the cut to be. Hold the dowel rod firmly on a table top or on the floor, and press the blade of a utility knife into the mark. You do not need to move the blade in a sawing motion, and you do *not* want to try to press it all the way through the dowel. You just need to cut a groove in it. Roll the dowel slightly and make another groove in line with your first, and repeat the process until there is one groove all the way around the dowel. Then pick up the dowel, and holding it with your thumbs on either side of the groove, make a quick snapping motion to break the dowel.

To hang a box from the dowel, punch a hole in the box and run a piece of thread through the hole. Then knot the thread well. Simply taping the thread to the box, instead of tying it through a hole, will not work -- at least not for long. The other end of the thread should be tied to the dowel. Again, simply taping the thread to the dowel may or may not hold up over time, and it generally doesn't look very good.

Once all four components of your mobile are in place and the mobile is well balanced so that the dowel rods are horizontal rather than slanted downward, check to make sure that the different components of the mobile will not bump into each other or get tangled up when the mobile turns. When you are satisfied with your mobile's appearance, put a small dab of white glue or clear nail polish on every knot. This will make sure they stay in place over time -- even if you drop your mobile or some other disaster befalls it.

Finally, trim off any extra pieces of thread hanging down from your knots, and you should be ready to hang your mobile from the ceiling.

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