Measurement and Variation

Synopsis

Students make careful observations in order to make individual identifications and also to explore the range of variation in a particular sample of biological structures (peanuts). When trying to measure variation, scientists are often confronted with special problems. When they make their measurements, variations may arise that are caused by the *measuring tools*. Or sometimes variations may arise due to the foibles of the *measurers*! Students will explore these problems as well.

Objectives

This exercise emphasizes the concept of *variation*. First, it calls attention to the natural variation that exists between individuals of a single species, which is one of the objectives of *NSES Content Standard C (Life Sciences)* for levels 5-8. Then, it asks students to measure that variation. In the course of doing so, students discover another important meaning of the word variation: the fact that two scientists might measure the same phenomenon using the same tools, but obtain different results. The learning objectives for this exercise, therefore, also include the following abilities necessary to do scientific inquiry from the *NSES Content Standard A (Science as Inquiry)* for levels 5-8:

- students will be able to use appropriate tools and techniques to gather, analyze, and interpret data; and
- students will be able to use mathematics in scientific inquiry.

Introduction

No two human beings look exactly alike. Not even identical twins. But how about raccoons? Or goldfish? How hard is it for us to tell individuals apart? How hard is it for *them* to tell individuals apart? Actually, we know, based on an enormous amount of empirical data, that there is lots of variation in nature, and we can see it if we look hard enough. This variation is the basis for evolution. Being bigger or smaller, faster or slower, producing seeds earlier or later--these ranges of characteristics are the fodder for natural selection. Try to imagine an alpine meadow at high elevation, covered with wildflowers. If we picked a particular species, say a beautiful blue alpine lupine, and kept track of the life cycle of individual plants, we would see something interesting. In any given summer, a few plants will flower very early, a few plants will flower very late, and the rest will all flower in some middle span of days or weeks. When these plants set seed, the vast majority of seeds for the next generation will come from that middle group. If there is a genetic component to this timing, most of next year's plants will also be 'middlers.'

But what if there is a very late, killing frost one particular year? This may stop the flowering of all the plants except the few late ones. Next year's crop will now come almost exclusively from these 'lupine-come-latelies.' If this late frost is actually an indication of long-term climate change, then there will be similar frosts in ensuing years, and over a long period of time, the genetic make-up of the entire population will gradually shift to favor the late-flowering individuals. Conversely, if drought were the issue, resulting in mid-summer fires, we could imagine a long-term shift favoring early-flowering individuals. Thus the characteristics in a population ebb and flow, reflecting adaptations to changes in the environment. Evolution.

In order to look at variation in the classroom setting, it will be easier to look at something that students are both familiar with and can easily deal with. Peanuts are both cheap and plentiful, familiar and tasty. (You might want to ask about allergies, however. Some people are quite sensitive to peanuts.) Since humans have been cultivating peanuts for a long time, many different varieties are available. Some tend to have two peanuts per shell, while others have three, four, or even five. Some have very large peanuts; others are quite small. But they all have a particularly characteristic growth pattern and life cycle.

The seeds (individual peanuts) germinate and grow to about a foot tall when they begin to flower. The tiny bright yellow flowers (at least in the varieties we have grown) appear in the *axils*, the place where the leaves grow out of the main stems. After the flowers fade, a peculiar structure called a *peduncle* begins to grow from the axil at the base of the old flower. This peduncle looks like a stem, and as it grows longer it begins to droop down. Eventually, it touches the ground and then actually *grows into the ground*. At this point a tiny peanut pod begins to grow at the tip of the peduncle; thus, the peanuts grow underground, but form on a structure that comes from above the ground. (The peanuts *do not* grow from the roots.) In other words, you can think of peanuts as being like peas or beans (to which they are closely related) that form their pods underground.

Another interesting thing about peanuts is that, being *legumes*, they have the ability to associate *symbiotically*, with nitrogen-fixing bacteria. Plants cannot get the essential element nitrogen directly from the air, even though nitrogen gas makes up almost 80% of the atmosphere. However, there are certain bacteria that live in the soil that can take atmospheric nitrogen and convert it to nitrites and nitrates, a process called nitrogen-fixation. These latter chemicals can be used by plants. Peanuts, and other legumes like peas and beans and clover, take this one step further. They actually 'allow' these nitrogen-fixing bacteria to set up shop in their roots. As a result, the peanut plant gets the direct benefit of having the usable nitrogen compounds right there in its roots. And presumably, the bacteria derive some benefit from the peanut as well--nutrients, water, safety. If you dig up a mature peanut plant, look carefully at the roots (not the peduncles). Usually, there will be some round lumps, called *nodules*, scattered here and there along the roots. This is where the bacteria live.

This relationship between peanuts and nitrogen-fixing bacteria also has its benefits for humans. It turns out that the bacteria fix so much nitrogen that they actually fertilize the soil with all the excess. George Washington Carver, the director of agricultural research at Tuskegee Institute during the first half of this century, realized that if farmers planted peanuts in their fields in rotation with other crops, they could take advantage of this free nitrogen fertilizer. Carver convinced farmers to use peanuts in a large variety of ways, and in the process, those farmers became better stewards of their soil.

Procedures

FIND YOUR PEANUT:

[This is a shortened version of our exercise **Find Your Peanut**. You might want to do that exercise instead.]

Give each student a peanut to study carefully. Ask them not to mark their peanuts in any way. If your students keep a science notebook, you might have them sketch their peanuts or describe them in writing. After a few minutes, put all the peanuts together in a bag, mix them up, pour them out with a flourish, and ask them to find their own peanut. The follow-up discussion might touch on issues like:

- What characteristics were useful for distinguishing individuals?
  [Size, shape, one bulge a lot bigger than the other, cracks or marks on the surface, etc.]
- Were there some characteristics that were not so useful for this exercise but that might be significant in the life of the peanut?
  [Overall comparative length might be difficult to distinguish on visual inspection, but longer peanuts might indicate larger seeds, and larger seeds might indicate more stored food for the developing embryo when the seed germinates.]
- Where was the peduncle attached to the peanut?
  [Toward one end, there is a spot with fiber lines radiating from it. It is generally easier to crack open a peanut at the *other* end.]
- If you want to break them open, you could discuss the various parts of the peanut: the pod is actually ovary tissue from the parent plant, the papery brown cover is the seed coat and is made of cellulose (fiber), most of the edible part is endosperm (the triploid, food storage part of the seed, filled with fats and carbohydrates), and the germ at one end is the embryo (the baby that might become a new plant if the seed were put in soil instead of roasted and eaten).

MEASUREMENT:

This exercise and the next explore the variation among peanuts in two ways--the actual variation in length of different peanuts, and the variation in measurements that *different* people get when they measure the *same* peanuts.

Give each student 15-20 peanuts and a ruler. If you give them roasted peanuts, they can eat them when they are finished. We suggest that you go through the peanuts beforehand to get rid of broken or shriveled shells. Ask the students to measure and record the maximum length of each peanut, to the nearest millimeter (or tenth of a centimeter, if you want them to experience decimals). This is not as trivial as it may seem. The maximum length may lie along a diagonal. And there will be parallax problems, depending on where the student places the ruler with respect to the peanut.

We like to have our students keep science notebooks, and we ask them to keep their data there. This gives them the opportunity to figure out how to organize their data so they are clear and retrievable. In addition to using their notebooks for gathering and working with their data, they can also use them for speculating on some of the questions raised below, recording other students' speculations, or adding a few sentences on what they learn from this whole process. However, if you prefer, you may copy and pass out the data forms attached to this exercise.

Once the students have gathered their data, you could ask them to find the following:

- ***mean***: Add up their individual measurements and divide by the number of peanuts. This will give them the average size of the peanuts in their sample.
- ***median***: Arrange their measurements from shortest to longest, and then find the length of the middle peanut.
- ***mode***: Record the most common length in their sample. In some cases there may not be a mode--i.e., all the peanuts are different lengths. In other cases there might be more than one mode.
- ***range***: Find the shortest and longest peanuts in their samples.

These numbers can lead to discussions about variation.

1. Write all the means on the board. How do they compare? What about the *mean of the means*? Is this a good estimate of the *average* peanut? [*Yes*, for the peanuts measured in this class that all came from the same bag in a particular market. *Maybe not*, for all the peanuts in the world, since there are so many different varieties grown.]
2. How does the *median* compare to the *mean*? What is the effect of one very large peanut on these two numbers?
   [For samples of 15-20, one large peanut might effect the *mean*, but should not have much of an effect on the median. Imagine a small business with one president and 10 employees. One highly paid executive in a company might shift the *mean* significantly, but the median will probably stay the same. If the executive makes $200,000 while each of ten employees makes $20,000, the *mean* salary would be in the neighborhood of $36,000 while the *median* would be $20,000. The executive salary makes the *mean* look pretty good. But that executive is only one person and does not shift the median at all.]
3. The *mode* probably won't indicate much for each student's sample. However, it might be interesting to pool all the data for a class and then look for the mode. [See below.] How does it relate to the *mean* or the *median*?
4. Does the *range* for each student or for the whole class tell you something about peanuts? Are all peanuts close to the *mean* in size or do they vary widely from the *mean*? [See below.]

Finally, you might want to pool all the data from your class.

#	length	#	length	#	length	#	length	#	length
1	17	21	28	41	34	61	36	81	39
2	18	22	28	42	34	62	37	82	40
3	18	23	29	43	34	63	37	83	40
4	19	24	29	44	35	64	37	84	40
5	20	25	29	45	35	65	37	85	40
6	20	26	29	46	35	66	37	86	40
7	20	27	29	47	35	67	37	87	41
8	20	28	30	48	35	68	37	88	41
9	20	29	30	49	35	69	37	89	42
10	20	30	30	50	35	70	37	90	42
11	21	31	30	51	35	71	38	91	42
12	21	32	31	52	35	72	38	92	45
13	21	33	31	53	36	73	38	93	45
14	25	34	31	54	36	74	38	94	45
15	25	35	31	55	36	75	38	95	46
16	25	36	32	56	36	76	38	96	46
17	26	37	34	57	36	77	39	97	46
18	27	38	34	58	36	78	39	98	47
19	27	39	34	59	36	79	39	99	51
20	27	40	34	60	36	80	39	100	56

Figure 1

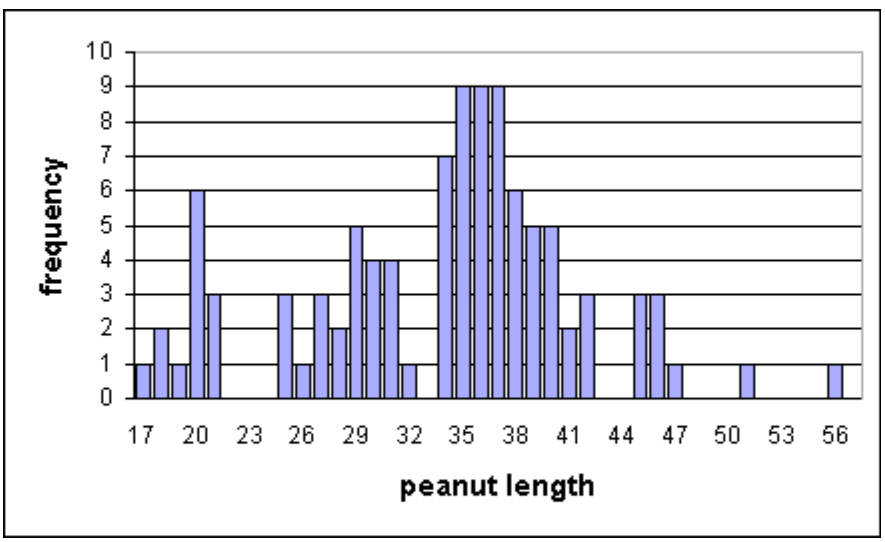

Figure 2

Figure 1 is a table of lengths of real peanuts we bought at the grocery store. These data have not been doctored in any way, and therefore might be similar to the results you will get with your students, though of course they did not come out of the bag in this particular order. We have arranged the data so that the peanuts are in order from smallest to largest (the beauty of computer spreadsheets). Figure 2 is a *histogram* (bar graph), showing the frequency of occurrence of the various peanut sizes. Here is a summary of the values we have been talking about.

mean	***median***	***mode***	***range***
33.8	***35***	***35, 36, 37***	***17 to 56***

The *mean* was determined by adding up all the lengths and dividing by the number of peanuts. The *median* was easy to find. Just look at the middle peanut in the table--the 50th peanut out of 100 (or split the difference between #50 and #51 if they are not the same length). Note that the *mean* and the *median* are almost the same. The *mode* was determined by looking at the graph and seeing that the most common lengths were 35, 36, and 37. Again, note that the *mode* is similar to the *mean* and the *median*. However, note that there is a cluster of values at the left end of the graph, with a small *mode* at 20 mm. This cluster is followed by a gap. These peanuts all had just one nut inside.

The *range* is from the smallest peanut to the largest. Note that most of the peanuts are in the center of this graph. There are fewer peanuts at either of the extremes of the entire range. In fact, if we were to draw a line through the tops of the bars and then smooth that line to a gentle curve, we would have a pretty good approximation to a *normal*, or *bell-shaped*, curve. This is typical of values found when measuring biological populations. A graph of the alpine lupines we discussed earlier would probably look very similar. That is, we would find very few early bloomers, most would flower in the mid-range, and only a few would be late bloomers. Accurate measurements of robin bills, salamander toes, tree buds, and 13 year old humans will all yield similar results.

THE HUMAN FACTOR:

This exercise can be done concurrently with the previous exercise or during some other classroom activity. Each student in the class will measure the lengths of the *same* sample of peanuts. The results will be tabulated and analyzed to look for differences that are the result of the ways humans measure objects.

Choose 20 typical peanuts from your source supply. Number them, using a permanent marker. Ask students to write the numbers 1 to 20 in a column on a piece of paper. Finally, ask each student to measure and record the *maximum length* of each of the peanuts. Again, they can record the data in their science notebooks or on the worksheet at the end of this exercise. They should use a metric ruler and round off their measurements to the nearest millimeter. They should also calculate their *mean* for these data. Each student should measure the same 20 peanuts, so it is best to keep the class involved in other activities while this is going on. You could also set this up such that the peanuts are passed around the room continuously. After all the students are finished, collect their data.

peanut number	Norm1	Norm2	Steve	Julie	Shelley	mean for each peanut (rows)	measured with callipers
	person measuring						
1	33	33	34	36	35	34.5	34.1
2	39	39	40	41	41	40.3	39.9
3	33	34	35	35	35	34.8	33.9
4	34	35	36	37	37	36.3	35.4
5	36	36	37	38	38	37.3	36.7
6	37	37	38	39	39	38.3	38.0
7	38	39	40	40	42	40.3	40.5
8	37	36	38	39	39	38.0	37.5
9	34	34	35	35	36	35.0	34.8
10	41	41	42	44	43	42.5	42.2
11	38	39	41	41	41	40.5	40.6
12	44	44	46	46	48	46.0	45.0
13	31	32	32	32	33	32.3	32.2
14	36	35	37	38	38	37.0	36.9
15	35	35	36	38	37	36.5	36.2
16	39	39	40	42	41	40.5	40.2
17	37	38	39	40	40	39.3	38.3
18	39	39	40	41	41	40.3	40.5
19	43	43	45	46	45	44.8	44.0
20	38	38	39	40	40	39.3	39.9
mean for all 20 peanuts (columns)	37.1	37.3	38.5	39.4	39.5	**38.7**	38.3

Figure 3

Figure 3 shows actual data we gathered. Again, we have not doctored the data in any way, so your results may be similar to ours. Norm measured the sample twice (Norm 1 and Norm 2), on two different days. The right-hand column shows measurements made using precision calipers rather than a simple plastic ruler. For demonstration purposes, we have arranged the means from smallest to largest (left to right).

Note that Norm did record different values for several of the peanuts in the sample, but the *means* of his two samples were very similar. Steve consistently recorded larger values for these peanuts. Most of his measurements were a millimeter larger than those made by Norm. *None* of his measurements was smaller! This resulted in a higher mean value. Julie and Shelley, though similar to each other, were consistently higher than either Norm or Steve.

What is the source of these differences? Some possibilities:

- One person may line up the left edge of a peanut with the 0 mark on the ruler, holding his eye directly above that mark. He then shifts his head so that his eye now looks straight down on the right edge of the peanut. Another person, after lining up the left edge, may simply look to the right without shifting her head. This difference in parallax may result in a different (larger) measurement.

- Some people forget that the end of a ruler may be in pretty bad shape, with edges gouged, scraped, or broken. Or the 0 mark may be set in from the edge of the ruler. If the peanut is lined up with the edge rather than a specific mark, a different reading will result.
- Rounding off. Should rounding be done by trying to judge whether the edge of the peanut is more than halfway to the next mark on the ruler? Or when we are not sure, should we round half up and half down just to be 'fair?'

The measurements made with calipers generally seem closest to those made by Steve, and the means are very close. Are caliper measurements better than those made with a ruler? Do these measurements tell us that Steve did a better job than Norm, Julie, or Shelley? A good set of calipers is a nice tool to use, but it can introduce unforeseen problems. When adjusting the jaws of the calipers to span the peanut, it is very easy to squeeze the peanut and get a shorter measurement. How much squeezing is appropriate? This can be difficult to judge. (By the way, to keep the record straight, it should be noted that Norm made the caliper measurements on a third day, so that he could not remember his numbers from his previous measurements. Many scientists are very picky about such things!)

In one of our summer workshops, we asked our students to measure the lengths of peanuts as described in the *Measurement* section above. 32 middle school students measured 20 peanuts each, for a total of 640 peanuts. Figure 4 below is a graph of their results. (Compare this to the graph in Figure 2 above.)

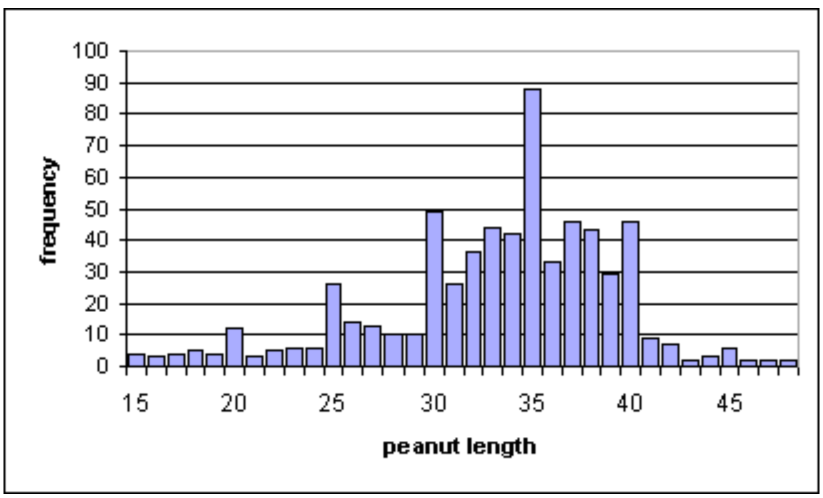

Figure 4

Although this graph shows a fairly typical normal distribution (bell-shaped curve), something very striking shows up when the graph is examined carefully. Note that peanut sizes of 15, 20, 25, 30, 35, 40, and 45 are all *more common* than other sizes. Apparently, enough of the 32 students *rounded off* their measurements to the nearest 5 millimeters to skew the results. Did they not understand the directions about rounding to the nearest millimeter? Did they round to the nearest long line on the ruler instead? Curiously, a second group of 32 students did exactly the same thing a month later!

Summary

So how long are these peanuts? Scientists are often confronted with situations like these. When they make measurements, can they really compare their results to those of other scientists who have come before them? Can they rely on their own consistency? How much practice does it take before they can feel secure in the reliability of their own techniques?

When looking for the effects of some experimental technique, a scientist may need to make measurements before and after treatment, or on treated and untreated (control) samples. For example, in testing a fertilizer, a scientist may need to measure the growth of plants. The variations in the plants themselves must be taken into consideration, as well as any differences due to measuring techniques. The design of the experiment must account for these difficulties. Careful technique can minimize the human inconsistencies. Statistical analysis can help with the inherent variability of the plants themselves. But that is for another exercise.

Special Note

After *observation*, *measurement* is probably the most important and widely used scientific technique. If measurement is fraught with pitfalls as shown, what is scientific TRUTH? Just how long *are* peanuts anyway? Is this one reason science sometimes gets bashed in the press--because a scientist hesitates to give a precise answer to a question?

Instructions for Students and Student Worksheet

No two human beings look exactly alike. Not even identical twins. But how about raccoons? Or goldfish? How hard is it for us to tell individuals apart? How hard is it for *them* to tell individuals apart? Biological variation is the basis for evolution. Being bigger or smaller, faster or slower, producing seeds earlier or later--these ranges of characteristics are the fodder for natural selection. But recognizing and measuring these differences can be trickier than it might seem.

MEASUREMENT:

This exercise and the next explore the variation among peanuts in two ways--the actual variation in length of different peanuts, and the variation in measurements that *different* people get when they measure the same peanuts.

Your teacher will give you 15-20 peanuts and a ruler. Please measure and record the maximum length of each peanut, to the nearest *millimeter*, on the attached data sheet. This is not as trivial as it may seem. The maximum length may lie along a diagonal. And there may be parallax problems, depending on where you place the ruler with respect to the peanut, and how you position your head when you take the measurement. After you have gathered your data, find the following:

- ***mean***: Add up your individual measurements and divide by the number of peanuts. This will give you the average size of the peanuts in your sample.
- ***median***: Arrange your measurements from shortest to longest, and then find the length of the middle peanut.
- ***mode***: Record the most common length in your sample. In some cases there

may not be a mode--i.e., all the peanuts are different lengths. In other cases there might be more than one mode.

- ***range***: Find the shortest and longest peanuts in your sample.

Please record these numbers on the attached data sheet as well. Turn in your results to your teacher after you have finished the next section.

THE HUMAN FACTOR:

In this exercise, you will measure the same 20 peanuts as everyone else in your class. Your teacher will pass around a container with peanuts marked with the numbers 1 through 20. Measure each one and record its length next to its number on your data sheet. Also calculate the mean for your data. Don't discuss your results with other students till everyone is finished. The results may surprise you.

Would you go up in the space shuttle if you were responsible for measuring important engine parts?

Measurement		
peanut measurements		ordered from small to large
mean		
median		
mode		
range		

The Human Factor	
#	peanut measurements
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
mean	

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