Fill the Number Grid

Objectives

The following number game is an excellent opening exercise for a whole class to tackle. It is a discovery exercise in which all students can participate. As the grid is filled in, some patterns will emerge which can be seen even by students who consider themselves 'weak' in math. Thus, everyone in the class gets a chance to find a correct answer. But the actual rule for filling in the grid also yields some results which may prove challenging to even the math 'whizzes' in the class. [We will not reveal the rule for filling in the grid till later in the **Procedure** section, in case you would like to figure out the rule yourself first.]

Procedure

Present students with the following grid written on the chalkboard. Their job is to fill in the grid.

12												
11												
10							70					
9												
8												
7												
6												
5				20								
4												
3								24				
2												
1												
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

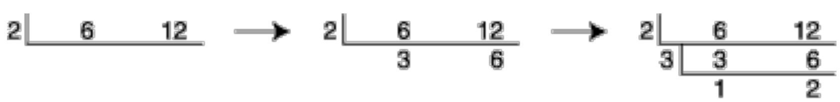
The ground rules for this game are similar to those for **What's the Rule?** Students make suggestions for numbers they would like to place in the grid. The grid is set up like a set of coordinate axes, so students name the box they would like to fill by saying the 'x' number first, followed by the 'y' number. For example, box (4,5) has already been filled with the number 20. This gives students practice for naming coordinates in a standard x,y graph. If a student guesses a number correctly, that number is placed in the grid. We make the rule that only students who raise their hands get to put numbers in the grid. Students who just call out numbers are ignored. In this way, the teacher can 'reward' specific students by allowing them to fill in several numbers in a row once they find a specific pattern. It also makes it possible to get more students involved, rather than letting a few aggressive students dominate the action.

The three numbers that have already been placed in the grid are a bit misleading. They have been chosen to make it look like this is just a table of multiplication facts. However, if you look at the completed grid below, you will quickly see that this is not the case.

12	12	12	12	12	60	12	84	24	36	60	132	12
11	11	22	33	44	55	66	77	88	99	110	11	132
10	10	10	30	20	10	30	70	40	90	10	110	60
9	9	18	9	36	45	18	63	72	9	90	99	36
8	8	8	24	8	40	24	56	8	72	40	88	24
7	7	14	21	28	35	42	7	56	63	70	77	84
6	6	6	6	12	30	6	42	24	18	30	66	12
5	5	10	15	20	5	30	35	40	45	10	55	60
4	4	4	12	4	20	12	28	8	36	20	44	12
3	3	6	3	12	15	6	21	24	9	30	33	12
2	2	2	6	4	10	6	14	8	18	10	22	12
1	1	2	3	4	5	6	7	8	9	10	11	12
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

The numbers in the grid are the *Least Common Multiples* (LCM) of the bold face numbers along the bottom and left axes. That is, 18 is the smallest number that both 6 and 9 will go into evenly. 18 is also the smallest number that 2 and 9 will go into evenly.

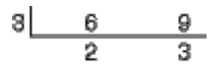
Example 1: You can get these least common multiples by using a technique that is sort of a repeated upside-down division. Example 1 shows this technique for finding the least common multiple for the numbers 6 and 12:



Example 1

First, divide both numbers by the smallest prime number (2) that will go into both evenly. Write down the quotients 3 and 6 underneath 6 and 12, respectively. Then divide both of those numbers by the smallest prime number (3) that will go into *them* evenly. Again write down 1 and 2 under them. Now multiply the *divisors* and the last two *quotients* together to get the least common multiple: **(2)(3)(1)(2) = 12**. That is, 12 is the smallest number that both 6 and 12 will go into evenly.

Example 2: Here is another example, finding the LCM for 6 and 9:



Example 2

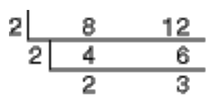
Only one division is possible. Therefore, the LCM is: **(3)(2)(3) = 18**.

Example 3: And here is an example for finding the LCM for 2 and 9:

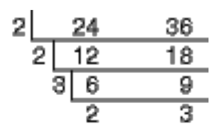
Example 3

Since there is *no* number that goes into *both* 2 and 9 evenly, simply multiply them together to get the LCM: **(2)(9) = 18**.

Examples 4 and 5: Here are two that require more steps to show that you just keep going till you cannot go any more:



Example 4



Example 5

So the LCM for 8 and 12 is **(2)(2)(2)(3) = 24**. And the LCM for 24 and 36 is **(2)(2)(3)(2)(3) = 72**.

As you can see in the grid, there are some interesting patterns. For example, the main diagonal from lower left to upper right is the numbers themselves. The left-hand column and the bottom row are all multiples of 1; therefore, they are also the numbers themselves. Rows and columns involving prime numbers (e.g., 7 and 11) *do* look like the simple multiplication facts, except for the *special cases* of (7,7) and (11,11). These *special cases*, of course, can be important clues to understanding the whole grid. Rows and columns involving non-primes (*e.g*., 4s, 6s, and 12s) are the ones that are surprising and/or confusing and, therefore, the most interesting.

Extensions

One teacher who used this exercise (Mary Hebrank, Duke School for Children) presented it a bit differently. She began the exercise as outlined above, but then, since her students did not finish it right away, she devised a poster with the grid on it. Students could come to her with their suggested numbers, and if they were correct, she would add them to the grid with post-it notes. The grid was finally filled after several days. Students were also encouraged to write out and submit their explanations of how the grid worked.

Revised: February 20, 2001